Question 6

A nonempty set S with a binary operation $*$ is called a semigroup if

i) $a, b \in S$, then $a * b \in S$

ii) $a * (b * c) = (a * b) * c$ for all $a, b, c \in S$

Furthermore, a nonempty set T with a ternary operation $\circ$ is called a ternary semigroup if

i) $a, b, c \in T$, then $a \circ b \circ c \in T$

ii) $(a \circ b \circ c) \circ d \circ e = a \circ (b \circ c \circ d) \circ e = a \circ b \circ (c \circ d \circ e)$ for all $a, b, c, d, e \in T$

Prove that S with the given operation $*$ is also a ternary semigroup.

(10 marks)

Solution:

Assume $(S, *)$ is a semigroup. Therefore S is closed and associative.		
i) Let $a, b, c \in S$, therefore		
$a * b * c = b' * c$	since S is closed; i.e. $a * b = b'$	
$\in S$	again since S is closed	
ii) $(a * b * c) * d * e = (a * b') * d * e$	since S is closed and by letting $b' = b * c$	
$= a * (b' * d) * e$	since S is associative	
$= a * (b * c * d) * e$	by replacing $b' = b * c$	
$= a * (b * c') * e$	since S is closed and by letting $c' = c * d$	
$= a * b * (c' * e)$	since S is associative	
$= a * b * (c * d * e)$	by replacing $c' = c * d$ ■	